

Comment about the "Gravity coupled to a scalar field in extra dimensions" paper.

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Wehus and Ravndal¹ have argued that in $d + 1$ dimensions the general solution for gravity minimally coupled to a scalar field can not be explicitly written in Schwarzschild coordinates. We contest these objections.

There has been a revival of interest in scalar fields theory in recent times, particularly in the context of string theory. The continuing focus on extra-dimensional models in fundamental physics provides motivation for studying scalar fields in higher dimensions. The physical relevance of scalar fields in today's gravitational physics and cosmology also stems from their role in current cosmological models². It is important to note in this connection that the widely employed equivalence between $(D+1)$ Kaluza-Klein theories with empty D -dimensional Jordan Brans-Dicke theories ($\omega = 0$) lead us to considerations a scalar field. Thus in this sense dimensional reduction "generates" sources which in lower dimension and can be interpreted as scalar field.

In a recent paper, Wehus and Ravndal¹, provide a good overview of the various Einstein-scalar field solutions and suggested that the general solution of the equations for gravity coupled minimally to a massless scalar field can not be explicitly written in Schwarzschild coordinates. The purpose of this short comment is to demonstrate that these solutions can be alternatively derived by exploiting the change of variables technique. First time, this technique was applied to a spherically symmetric case for finding a new solution to the Jordan Brans-Dicke-scalar field equations³. Techniques for obtaining the similar static solutions are known by know⁴. In $D = d + 1$ dimensions, the action for a scalar field minimally coupled to gravity given by (we take units $G = c = 1$):

$$S = \int d^D \sqrt{-g} \frac{1}{2} (R - g^{\mu\nu} \phi_{,\mu} \phi_{,\nu}). \quad (1)$$

One can get the minimally coupled Einstein-scalar fields equations by variation the above action:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \left(\nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi \right), \quad (2)$$

$$\phi = 0. \quad (3)$$

∇_α is the covariant derivative associated with the metric g , $R_{\mu\mu}$ and R are the Ricci tensor and Ricci scalar for an arbitrary metric g .

By contracting equation (1), we can rewrite this equation as

$$R_{\mu\nu} = \nabla_\mu \phi \nabla_\nu \phi, \quad (4)$$

As we have already mentioned we consider standard static and spherically symmetric space-time. The static and spherically symmetric metric in Schwarzschild coordinates for $d + 1$ dimensions can be written as

$$ds^2 = -e^{2\alpha(r)} dt^2 + e^{2\beta(r)} dr^2 + r^2 d\Omega_d^2. \quad (5)$$

where α, β are unknown functions of the radial coordinate r , and $d\Omega_d^2$ is the solid angle element in $d - 1$ dimensions. In order to simplify the problem of solving the field equations we will replace variable r by $r(\alpha)$ then the field equations (4) take a form:

$$1 - \left(1 + (d-1) \frac{r'(\alpha)}{r(\alpha)}\right) \lambda'(\alpha) - \frac{r''(\alpha)}{r'(\alpha)} = -\phi'(\alpha)^2, \quad (6)$$

$$-1 + (d-2) \left(e^{2\lambda(\alpha)} - 1\right) \frac{r'(\alpha)}{r(\alpha)} + \lambda'(\alpha) = 0, \quad (7)$$

$$-1 + (d-1) \frac{r'(\alpha)}{r(\alpha)} - \frac{r''(\alpha)}{r'(\alpha)} - \lambda'(\alpha) = 0, \quad (8)$$

and the equation of motion for the scalar field (3)

$$-1 + (d-1) \frac{r'(\alpha)}{r(\alpha)} - \frac{r''(\alpha)}{r'(\alpha)} - \lambda'(\alpha) + \frac{\phi''(\alpha)}{\phi'(\alpha)} = 0, \quad (9)$$

where α is a new variable and the primes denote derivatives with respect to α . Making use of equation (8) in (9) they simplify to

$$\frac{\phi''(\alpha)}{\phi'(\alpha)} = 0, \quad (10)$$

thus we obtain

$$\phi(\alpha) = \zeta + \xi\alpha, \quad (11)$$

where ζ and ξ is a arbitrary constants. Using the asymptotic condition in infinity we have $\zeta = 1$. In the case $\xi = 0$ one can find solution of equations (6) - (8)

$$r(\alpha) = \frac{\text{const}}{e^{2\alpha} - 1}, \quad e^{\lambda(\alpha)} = e^{-\alpha}, \quad \phi(\alpha) = 1, \quad (12)$$

that identical to the Schwarzschild solution of the Einstein theory. For the more general case $\xi \neq 0$ making use equations (6), (8) and (11) we eliminate $\lambda'(\alpha)$ and obtain for $r(\alpha)$

$$r(\alpha) = e^{-\frac{\alpha}{d-2}} \chi \cosh \left(\sqrt{1 + \frac{(d-2)\xi^2}{d-1}} (\alpha + \psi) \right)^{\frac{1}{d-2}}, \quad (13)$$

where χ and ψ is a arbitrary constants. After same algebra one can find from (7) - (8) for $\lambda(\alpha)$:

$$\begin{aligned} \lambda(\alpha) = & -\frac{1}{2} \log(d-2) - \log(r'(\alpha)) - \\ & -\frac{1}{2} \log(r(\alpha) + (d-1)r'(\alpha)) + \\ & +\frac{1}{2} \log[(2d-3)r(\alpha)r'(\alpha) + (d-2)(d-1)r'(\alpha)^3 + \\ & +r(\alpha)^2 r''(\alpha) - r'(\alpha)\xi^2], \end{aligned} \quad (14)$$

In the search for static and spherical symmetric solutions, we choose the metric in Schwarzschild coordinates. This gives us both Schwarzschild solution and more general solution of the minimally coupled equations. A point to be noted is that in the Jordan, Brans-Dicke theory this change of variables technique easily give us the arbitrary dimension Heckmann solution, too.

References

1. I. K. Wehus and F. Ravndal (2006). Gravity coupled to a scalar field in extra dimensions. Preprint arXiv:gr-qc/0610048v2
2. K.Tangen (2007). Generating Minimally Coupled Einstein-Scalar Field Solutions from Vacuum Solutions with Arbitrary Cosmological Constant. Preprint arXiv:gr-qc/ 0705.4372 v.2
3. P.Jordan , "Schwerkraft und Weltall", Braunschweig, (1955).
4. S. Kozyrev (2002). Properties of the static, spherically symmetric solutions in the Jordan, Brans-Dicke theory, Preprint arXiv:gr-qc/0207039 .